Appendix: Explanation of Formula for Predicting Quarantine Failure Rates

Let $L$ be a random variable denoting the incubation period, and $f(l)$ and $F(l)$ be its probability density function (p.d.f.) and cumulative distribution function. Let $M$ be a random variable denoting the maximum incubation period in a sample of $n$ infections. The p.d.f. of $M$ is then given by $n f(m) F(m)^{n-1}$. In other words, the probability that, after $n$ draws the maximum incubation period is $m$, is given by the product of the probability that one draw yields an incubation period of exactly $m$ (i.e., $nf(m)dm$) with the probability that the remaining $n-1$ draws all yield incubation periods no larger than $m$ (i.e., $F(m)^{n-1}$). Now introduce a new random variable, $X = F(m)$ (lying in $[0,1]$), representing the probability that an infected host will have an incubation period no larger than $m$ (where $F$ is the same cumulative distribution function introduced above). The p.d.f. of $X$ is

$$
\frac{d}{dx} \int_0^x n f(m) F(m)^{n-1} dm = nx^{x-1},
$$

where $G(x)$ is defined to be the inverse of $F(x)$. The quarantine failure rate is $1-X$, and therefore its p.d.f., $p(\phi)$, is

$$np(1-\phi)^{n-1}. \quad \text{We then also have} \quad X = \int_0^\pi n(1-\phi)^{x-1} \, d\phi = 1 - (1 - x)^{n} \quad \text{and} \quad \bar{\phi} = \int_0^1 n(1-\phi)^{x-1} \, d\phi = 1/(x + 1).$$