Modeling Insights into *Haemophilus influenzae* Type b Disease, Transmission, and Vaccine Programs

**Technical Appendix 1**

**Model Structure**

The following set of partial differential equations defines the rates at which the simulated population moves between model states:

\[
\frac{\partial NS}{\partial t} + \frac{\partial NS}{\partial a} = \mu(t,a)X(t) + \omega_L LS(t,a) + \omega_{BPIG} I(t,a) - \left( I(t) + \lambda(t,a) + \gamma(t,a)\varepsilon(a) + \delta_{BPIG} (t,a) \right) NS(t,a)
\]

\[
\frac{\partial I}{\partial t} + \frac{\partial I}{\partial a} = \delta_{BPIG} (t,a)NS(t,a) - \omega_{BPIG} I(t,a)
\]

\[
\frac{\partial NC}{\partial t} + \frac{\partial NC}{\partial a} = \lambda(t,a)NS(t,a) - \left( I(t) + \rho_c + \sigma(a) + \gamma(t,a)\varepsilon(a) \right) NC(t,a)
\]

\[
\frac{\partial LS}{\partial t} + \frac{\partial LS}{\partial a} = \omega_H HS(t,a) - \left( I(t) + \omega_L + \lambda(t,a)(1 - \alpha_L) + \gamma(t,a)\varepsilon(a) \right) LS(t,a)
\]

\[
\frac{\partial LC}{\partial t} + \frac{\partial LC}{\partial a} = \lambda(t,a)(1 - \alpha_L)LS(t,a) - \left( I(t) + \rho_c + \sigma(a)(1 - \beta_L) + \gamma(t,a)\varepsilon(a) \right) LC(t,a)
\]
\[
\frac{\partial HS}{\partial t} + \frac{\partial HS}{\partial a} = \rho_c \left[ C(t,a) + LC(t,a) + HC(t,a) - \rho_c D(t,a) + \gamma(t,a) \varepsilon(a) \right] S(t,a) + LS(t,a) - L(t) + \omega_H(a) + \lambda(t,a)(1 - \alpha_H) HS(t,a)
\]

\[
\frac{\partial HC}{\partial t} + \frac{\partial HC}{\partial a} = \lambda(t,a)(1 - \alpha_H) HS(t,a) + \gamma(t,a) \varepsilon(a) \left[ NC(t,a) + LC(t,a) \right] - \left[ H(t) + \rho_H + \sigma(a)(1 - \beta_H) \right] HC(t,a)
\]

\[
\frac{\partial D}{\partial t} + \frac{\partial D}{\partial a} = \sigma(a) \left[ C(t,a) + (1 - \beta_L) LC(t,a) + (1 - \beta_H) HC(t,a) \right] - \left[ D(t) + \rho_D \right] D(t,a)
\]

In which:

- NS, NC, LS, LC, HS, HC, D, and I are population states, where N=No antibody, L = Low antibody, H = High antibody, S = Susceptible, C = Colonized, D = Diseased, and I = Immune; X(t) is the total population.
- \( \mu(t,a) \) and \( \nu(t) \) are time-dependent birth and death rates, respectively. Birth rate also depends on age as individuals are only born into the age=0 group.
- \( \omega_L \) is the rate at which low antibody wanes to no antibody and \( \omega_H(a) \) is the age-dependent rate at which high antibody wanes to low antibody.
- \( \lambda(t,a) \) is the time- and age-dependent force of infection.
- \( \gamma(t,a) \) is the time- and age-dependent rate of vaccination, and \( \varepsilon(a) \) is the age-dependent vaccine take rate.
- \( \sigma(a) \) is the age-dependent rate of invasive disease among colonized persons.
- \( \alpha_L \) and \( \alpha_H \) are the efficacy of low and high antibody at preventing colonization.
- \( \beta_L \) and \( \beta_H \) are the efficacy of low and high antibody at preventing invasive disease.
- \( \rho_C \) and \( \rho_D \) are the rates of recovery from colonization and invasive disease, respectively.
- \( \delta_{BPIG}(t,a) \) is the time- and age-dependent rate of BPIG use (for Alaska Native populations only), and \( \omega_{BPIG} \) is the rate of waning of BPIG protection.