Modeling Insights into *Haemophilus influenzae* Type b Disease, Transmission, and Vaccine Programs

Technical Appendix 1

Model Structure

The following set of partial differential equations defines the rates at which the simulated population moves between model states:

\[
\frac{\partial NS}{\partial t} + \frac{\partial NS}{\partial a} = \mu(t,a) X(t) + \omega_L LS(t,a) + \omega_{BPIG} I(t,a) - \left\{ (t) + \lambda(t,a) + \gamma(t,a) \sigma(a) + \delta_{BPG} (t,a) \bar{NS}(t,a) \right\}
\]

\[
\frac{\partial I}{\partial t} + \frac{\partial I}{\partial a} = \delta_{BPG} (t,a) NS(t,a) - \omega_{BPG} I(t,a)
\]

\[
\frac{\partial NC}{\partial t} + \frac{\partial NC}{\partial a} = \lambda(t,a) NS(t,a) - \left\{ (t) + \rho_c + \sigma(a) + \gamma(t,a) \sigma(a) \bar{NC}(t,a) \right\}
\]

\[
\frac{\partial LS}{\partial t} + \frac{\partial LS}{\partial a} = \omega_H HS(t,a) - \left\{ (t) + \omega_L + \lambda(t,a)(1 - \alpha_L) + \gamma(t,a) \sigma(a) \bar{LS}(t,a) \right\}
\]

\[
\frac{\partial LC}{\partial t} + \frac{\partial LC}{\partial a} = \lambda(t,a)(1 - \alpha_L) LS(t,a) - \left\{ (t) + \rho_c + \sigma(a)(1 - \beta_L) + \gamma(t,a) \sigma(a) \bar{LC}(t,a) \right\}
\]
\[
\frac{\partial HS}{\partial t} + \frac{\partial HS}{\partial a} = \rho_C \left[ C(t,a) + LC(t,a) + HC(t,a) \right] - \rho_D D(t,a) + \gamma(t,a) \varepsilon(a) \left[ S(t,a) + LS(t,a) \right] - \left[ t + \omega_H(a) + \lambda(t,a)(1 - \alpha_H) \right] HS(t,a)
\]

\[
\frac{\partial HC}{\partial t} + \frac{\partial HC}{\partial a} = \lambda(t,a)(1 - \alpha_H) HS(t,a) + \gamma(t,a) \varepsilon(a) [ NC(t,a) + LC(t,a) ] - \left[ t + \rho_C + \sigma(a)(1 - \beta_H) \right] HC(t,a)
\]

\[
\frac{\partial D}{\partial t} + \frac{\partial D}{\partial a} = \sigma(a) \left[ C(t,a) + (1 - \beta_L) LC(t,a) + (1 - \beta_H) HC(t,a) \right] - \left[ t + \rho_D \right] D(t,a)
\]

In which:

- NS, NC, LS, LC, HS, HC, D, and I are population states, where N=No antibody, L = Low antibody, H = High antibody, S = Susceptible, C = Colonized, D = Diseased, and I = Immune; X(t) is the total population.
- \(\mu(t,a)\) and \(\nu(t)\) are time-dependent birth and death rates, respectively. Birth rate also depends on age as individuals are only born into the age=0 group.
- \(\omega_L\) is the rate at which low antibody wanes to no antibody and \(\omega_H(a)\) is the age-dependent rate at which high antibody wanes to low antibody.
- \(\lambda(t,a)\) is the time- and age-dependent force of infection.
- \(\gamma(t,a)\) is the time- and age-dependent rate of vaccination, and \(\varepsilon(a)\) is the age-dependent vaccine take rate.
- \(\sigma(a)\) is the age-dependent rate of invasive disease among colonized persons.
- \(\alpha_L\) and \(\alpha_H\) are the efficacy of low and high antibody at preventing colonization.
- \(\beta_L\) and \(\beta_H\) are the efficacy of low and high antibody at preventing invasive disease.
- \(\rho_C\) and \(\rho_D\) are the rates of recovery from colonization and invasive disease, respectively.
- \(\delta_{BPIG}(t,a)\) is the time- and age-dependent rate of BPIG use (for Alaska Native populations only), and \(\omega_{BPIG}\) is the rate of waning of BPIG protection.