A Dynamic Transmission Model for Predicting Trends in *Helicobacter pylori* and Associated Diseases in the United States

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Appendix

The mathematical equations underlying our compartmental model of *Helicobacter pylori* are a system of partial differential equations:

\[
\begin{align*}
\frac{dI}{dt} + \frac{dI}{da} &= -\mu(a) \cdot I(a,t) \\
\frac{dS}{dt} + \frac{dS}{da} &= -\left[\lambda_1(a,t) + \lambda_2(a,t) + \mu(a)\right] \cdot S(a,t) \\
\frac{dAG}{dt} + \frac{dAG}{da} &= \lambda_1(a,t) \cdot S(a,t) - \left[\delta_1(a) + \delta_2(a) + \mu(a)\right] \cdot AG(a,t) \\
\frac{dCG}{dt} + \frac{dCG}{da} &= -\lambda_2(a,t) \cdot S(a,t) - \left[\delta_3(a) + \mu(a)\right] \cdot CG(a,t) \\
\frac{dDU}{dt} + \frac{dDU}{da} &= \delta_4(a) \cdot AG(a,t) - \left[\delta_4(a) + \mu(a)\right] \cdot DU(a,t) \\
\frac{dCAG}{dt} + \frac{dCAG}{da} &= \delta_5(a) \cdot DUG(a,t) + \delta_4(a) \cdot CG(a,t) - \left[\delta_3(a) + \mu(a)\right] \cdot CAG(a,t) \\
\frac{dGC}{dt} + \frac{dGC}{da} &= \delta_5(a) \cdot CAG(a,t) - \left[\delta_3(a) + \mu(a)\right] \cdot GC(a,t)
\end{align*}
\]

The mathematical equations underlying our compartmental model of *H. pylori* is a system of partial differential equations:

\[
\begin{align*}
\lambda_1(a,t) &= p(a) \cdot \int_{a'} \beta(a',a) \cdot \left[AG(a',t) + CG(a',t) + DU(a',t) + \alpha \cdot CAG(a',t) + GC(a',t)\right] da' \\
\lambda_2(a,t) &= (1 - p(a)) \cdot \int_{a'} \beta(a',a) \cdot \left[AG(a',t) + CG(a',t) + DU(a',t) + \alpha \cdot CAG(a',t) + GC(a',t)\right] da'
\end{align*}
\]

where:
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\[ I(0,t) = p_I \cdot \Pi \]
\[ S(0,t) = (1 - p_I) \cdot \Pi \]
\[ AG(0,t) = CG(0,t) = DU(0,t) = CAG(0,t) = GC(0,t) = 0 \]

Notation:

\( a, a' \) age index
\( t \) time index
\( \Pi \) birth rate per unit time
\( I(a,t) \) number of isolated (not-susceptible) individuals of age \( a \), at time \( t \)
\( S(a,t) \) number of susceptible individuals of age \( a \), at time \( t \)
\( AG(a,t) \) number of infected individuals of age \( a \) with antrum-predominant gastritis, at time \( t \)
\( CG(a,t) \) number of infected individuals of age \( a \) with corpus-predominant gastritis, at time \( t \)
\( DU(a,t) \) number of individuals of age \( a \) with duodenal ulcer, at time \( t \)
\( CAG(a,t) \) number of individuals of age \( a \) with chronic atrophic gastritis, at time \( t \)
\( GC(a,t) \) number of individuals of age \( a \) with gastric cancer, at time \( t \)
\( p_I \) proportion of population that is not-susceptible at birth
\( \lambda_1(a,t) \) rate at which one susceptible of age \( a \) acquire infection and develop antrum-predominant gastritis
\( \lambda_2(a,t) \) rate at which one susceptible of age \( a \) acquire infection and develop corpus-predominant gastritis
\( \beta(a',a) \) transmission parameter; probability that an infective of age \( a' \) will infect a susceptible of age \( a \)
\( p(a) \) proportion of newly infected individuals of age \( a \) developing antrum (vs. corpus) predominant gastritis
\( \delta_1(a) \) transition rate from antrum- to corpus-predominant gastritis in age group \( a \)
\( \delta_2(a) \) progression rate from antrum-predominant gastritis to duodenal ulcer in age group \( a \)
\( \delta_3(a) \) transition rate from duodenal ulcer to chronic atrophic gastritis in age group \( a \)
\( \delta_4(a) \) progression rate from corpus-predominant gastritis to chronic atrophic gastritis in age group \( a \)
\( \delta_5(a) \) progression rate from chronic atrophic gastritis to gastric cancer in age group \( a \)
\( \mu(a) \) age-specific background mortality rate due to all cases
\( \mu_{DU} \) mortality rate due to duodenal ulcer
\( \mu_{GU} \) mortality rate due to gastric ulcer
\( \mu_{GC} \) mortality rate due to gastric cancer