

A Dynamic Transmission Model for Predicting Trends in *Helicobacter pylori* and Associated Diseases in the United States

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Appendix

The mathematical equations underlying our compartmental model of *Helicobacter pylori* are a system of partial differential equations:

$$\begin{aligned} \frac{\partial I}{\partial t} + \frac{\partial I}{\partial a} &= -\mu(a) \cdot I(a, t) \\ \frac{\partial S}{\partial t} + \frac{\partial S}{\partial a} &= -[\lambda_1(a, t) + \lambda_2(a, t) + \mu(a)] \cdot S(a, t) \\ \frac{\partial AG}{\partial t} + \frac{\partial AG}{\partial a} &= \lambda_1(a, t) \cdot S(a, t) - [\delta_1(a) + \delta_2(a) + \mu(a)] \cdot AG(a, t) \\ \frac{\partial CG}{\partial t} + \frac{\partial CG}{\partial a} &= \lambda_2(a, t) \cdot S(a, t) - [\delta_4(a) + \mu(a)] \cdot CG(a, t) \\ \frac{\partial DU}{\partial t} + \frac{\partial DU}{\partial a} &= \delta_2(a) \cdot AG(a, t) - [\delta_3(a) + \mu_{DU} + \mu(a)] \cdot DU(a, t) \\ \frac{\partial CAG}{\partial t} + \frac{\partial CAG}{\partial a} &= \delta_3(a) \cdot DU(a, t) + \delta_4(a) \cdot CG(a, t) - [\delta_5(a) + \mu_{CAG} + \mu(a)] \cdot CAG(a, t) \\ \frac{\partial GC}{\partial t} + \frac{\partial GC}{\partial a} &= \delta_5(a) \cdot CAG(a, t) - [\mu_{GC} + \mu(a)] \cdot GC(a, t) \end{aligned}$$

The mathematical equations underlying our compartmental model of *H. pylori* is a system of partial differential equations:

$$\begin{aligned} \lambda_1(a, t) &= p(a) \cdot \int_0^{\infty} \beta(a', a) \cdot [AG(a', t) + CG(a', t) + DU(a', t) + \alpha \cdot CAG(a', t) + GC(a', t)] da' \\ \lambda_2(a, t) &= (1 - p(a)) \cdot \int_0^{\infty} \beta(a', a) \cdot [AG(a', t) + CG(a', t) + DU(a', t) + \alpha \cdot CAG(a', t) + GC(a', t)] da' \end{aligned}$$

where:

$$I(0,t) = p_1 \cdot \Pi$$

$$S(0,t) = (1 - p_1) \cdot \Pi$$

$$AG(0,t) = CG(0,t) = DU(0,t) = CAG(0,t) = GC(0,t) = 0$$

Notation:

a, a'	age index
t	time index
Π	birth rate per unit time
$I(a,t)$	number of isolated (not-susceptible) individuals of age a , at time t
$S(a,t)$	number of susceptible individuals of age a , at time t
$AG(a,t)$	number of infected individuals of age a with antrum-predominant gastritis, at time t
$CG(a,t)$	number of infected individuals of age a with corpus-predominant gastritis, at time t
$DU(a,t)$	number of individuals of age a with duodenal ulcer, at time t
$CAG(a,t)$	number of individuals of age a with chronic atrophic gastritis, at time t
$GC(a,t)$	number of individuals of age a with gastric cancer, at time t
p_I	proportion of population that is not-susceptible at birth
$\lambda_1(a,t)$	rate at which one susceptible of age a acquire infection and develop antrum-predominant gastritis
$\lambda_2(a,t)$	rate at which one susceptible of age a acquire infection and develop corpus-predominant gastritis
$\beta(a',a)$	transmission parameter; probability that an infective of age a' will infect a susceptible of age a
$p(a)$	proportion of newly infected individuals of age a developing antrum (vs. corpus) predominant gastritis
$\delta_1(a)$	transition rate from antrum- to corpus-predominant gastritis in age group a
$\delta_2(a)$	progression rate from antrum-predominant gastritis to duodenal ulcer in age group a
$\delta_3(a)$	transition rate from duodenal ulcer to chronic atrophic gastritis in age group a
$\delta_4(a)$ □	progression rate from corpus-predominant gastritis to chronic atrophic gastritis in age group a
$\delta_5(a)$	progression rate from chronic atrophic gastritis to gastric cancer in age group a
$\mu(a)$	age-specific background mortality rate due to all cases
μ_{DU} □	mortality rate due to duodenal ulcer
μ_{GU}	mortality rate due to gastric ulcer
μ_{GC}	mortality rate due to gastric cancer

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