A Dynamic Transmission Model for Predicting Trends in Helicobacter pylori and Associated Diseases in the United States

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Appendix

The mathematical equations underlying our compartmental model of Helicobacter pylori are a system of partial differential equations:

\[
\frac{\partial I}{\partial t} + \frac{\partial I}{\partial a} = -\mu(a) \cdot I(a, t)
\]

\[
\frac{\partial S}{\partial t} + \frac{\partial S}{\partial a} = -[\lambda_1(a, t) + \lambda_2(a, t)] \cdot S(a, t) - [\delta_1(a) + \delta_2(a) + \mu(a)] \cdot A(a, t)
\]

\[
\frac{\partial AG}{\partial t} + \frac{\partial AG}{\partial a} = \lambda_1(a, t) \cdot S(a, t) - [\delta_1(a) + \mu(a)] \cdot AG(a, t)
\]

\[
\frac{\partial CG}{\partial t} + \frac{\partial CG}{\partial a} = \lambda_2(a, t) \cdot S(a, t) - [\delta_2(a) + \mu(a)] \cdot CG(a, t)
\]

\[
\frac{\partial DU}{\partial t} + \frac{\partial DU}{\partial a} = \delta_2(a) \cdot AG(a, t) - [\delta_2(a) + \mu DU + \mu(a)] \cdot DU(a, t)
\]

\[
\frac{\partial CAG}{\partial t} + \frac{\partial CAG}{\partial a} = \delta_3(a) \cdot DU(a, t) + \delta_4(a) \cdot GC(a, t) - [\delta_3(a) + \mu DU + \mu(a)] \cdot CAG(a, t)
\]

\[
\frac{\partial GC}{\partial t} + \frac{\partial GC}{\partial a} = -[\mu GC + \mu(a)] \cdot GC(a, t)
\]

The mathematical equations underlying our compartmental model of H. pylori is a system of partial differential equations:

\[
\lambda_1(a, t) = p(a) \int \beta(a', a) \cdot [AG(a', t) + CG(a', t) + DU(a', t) + \alpha \cdot CAG(a', t) + GC(a', t)] da'
\]

\[
\lambda_2(a, t) = (1 - p(a)) \int \beta(a', a) \cdot [AG(a', t) + CG(a', t) + DU(a', t) + \alpha \cdot CAG(a', t) + GC(a', t)] da'
\]

where:
Notation:

- $a, a'$: age index
- $t$: time index
- $\Pi$: birth rate per unit time
- $I(a,t)$: number of isolated (not-susceptible) individuals of age $a$, at time $t$
- $S(a,t)$: number of susceptible individuals of age $a$, at time $t$
- $AG(a,t)$: number of infected individuals of age $a$ with antrum-predominant gastritis, at time $t$
- $CG(a,t)$: number of infected individuals of age $a$ with corpus-predominant gastritis, at time $t$
- $DU(a,t)$: number of individuals of age $a$ with duodenal ulcer, at time $t$
- $CAG(a,t)$: number of individuals of age $a$ with chronic atrophic gastritis, at time $t$
- $GC(a,t)$: number of individuals of age $a$ with gastric cancer, at time $t$
- $p_I$: proportion of population that is not-susceptible at birth
- $\lambda_1(a,t)$: rate at which one susceptible of age $a$ acquire infection and develop antrum-predominant gastritis
- $\lambda_2(a,t)$: rate at which one susceptible of age $a$ acquire infection and develop corpus-predominant gastritis
- $\beta(a',a)$: transmission parameter; probability that an infective of age $a'$ will infect a susceptible of age $a$
- $p(a)$: proportion of newly infected individuals of age $a$ developing antrum (vs. corpus) predominant gastritis
- $\delta_1(a)$: transition rate from antrum- to corpus-predominant gastritis in age group $a$
- $\delta_2(a)$: progression rate from antrum-predominant gastritis to duodenal ulcer in age group $a$
- $\delta_3(a)$: transition rate from duodenal ulcer to chronic atrophic gastritis in age group $a$
- $\delta_4(a)$: progression rate from corpus-predominant gastritis to chronic atrophic gastritis in age group $a$
- $\delta_5(a)$: progression rate from chronic atrophic gastritis to gastric cancer in age group $a$
- $\mu(a)$: age-specific background mortality rate due to all cases
- $\mu_{DU}$: mortality rate due to duodenal ulcer
- $\mu_{GU}$: mortality rate due to gastric ulcer
- $\mu_{GC}$: mortality rate due to gastric cancer